HARD SCATTERING IN A NUCLEAR ENVIRONMENT: FAREWELL TO LINEAR $k_\perp$-FACTORIZATION *

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We discuss a dramatic change brought into the pQCD description of hard processes in a nuclear environment by a large thickness of heavy nuclei. It breaks the familiar linear $k_\perp$-factorization which must be replaced by a new concept of the nonlinear $k_\perp$-factorization introduced in [1].We demonstrate the salient features of nonlinear $k_\perp$-factorization on several examples from hard dijet production in DIS off heavy nuclei to single-jet to dijet production in hadron-nucleus collisions . We also comment briefly on the non-linear BFKL evolution for gluon density of nuclei.

1 Introduction

The linear $k_\perp$-factorization is a fundamental ingredient of the pQCD description of high energy hard processes off free nucleons. A large thickness of a target nucleus introduces a new scale - the so-called saturation scale Q_A^2, - which breaks the linear $k_\perp$-factorization theorems for hard scattering in a nuclear environment. This property can be linked to the unitarity constraints for the color dipole-nucleus interaction. In this talk we review the recent work by the ITEP-Jülich-Landau collaboration in which a new concept of the nonlinear $k_\perp$-factorization has been introduced [1, 2]. We illustrate this new concept on an example of dijet production in DIS off heavy nuclei and comment on more recent applications to single-jet [3] and dijet [4] production in pA collisions. The nonlinear $k_\perp$-factorization emerges as a generic feature of the pQCD approach to hard processes in nuclear environment, although the concrete realizations depend strongly on the relevant pQCD subprocesses.

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2 The $k_\perp$-factorization for DIS off free nucleons

The parton fusion description of the shadowing introduced in 1975 [5] is equivalent to the unitarization on the color dipole-nucleus interaction [6]. The starting point is the color dipole factorization for DIS at small $x \lesssim x_A = 1/R_A m_N$, when the coherency over the thickness of the nucleus holds for the $q\bar{q}$ Fock states of the virtual photon:

$$\sigma_T(x,Q^2) = \langle\gamma^*|\sigma(x,\mathbf{r})|\gamma^*\rangle = \int_0^1 dz \int d^2\mathbf{r}\Psi^*_{\gamma^*}(z,\mathbf{r})\sigma(x,\mathbf{r})\Psi_{\gamma^*}(z,\mathbf{r})\,. \tag{1}$$

Here z and $(1-z)$ is the energy partition between q & $\bar{q}$ and $\mathbf{r}$ = size of the color dipole. There is an equivalence between color dipole and $k_\perp$-factorization [6, 7, 8]:

$$\sigma(x,\mathbf{r}) = \alpha_S(r)\int \frac{d^2\boldsymbol{\kappa} 4\pi[1-e^{i\boldsymbol{\kappa}\mathbf{r}}]}{N_c\kappa^4}\cdot\frac{\partial G_N}{\partial\log\kappa^2}\,, \tag{2}$$

$$f(x,\boldsymbol{\kappa}) = \frac{4\pi}{N_c\sigma_0(x)}\cdot\frac{1}{\kappa^4}\cdot\frac{\partial G_N(x,\boldsymbol{\kappa})}{\partial\log\kappa^2}\,. \tag{3}$$

where $\sigma_0(x) = \sigma(x,\mathbf{r})\Big|_{r\to\infty}$. The x-dependence of $\sigma(x,\mathbf{r})$ is governed by the color dipole BFKL equation [9]. The unintegrated gluon density $f(x,\boldsymbol{\kappa})$ furnishes a universal description of $F_{2p}(x,Q^2)$ and of the final states. For instance, the linear $k_\perp$-factorization for forward dijet cross section reads

$$\frac{(2\pi)^2 d\sigma_N}{dz d^2\mathbf{p}_+ d^2\boldsymbol{\Delta}} = \frac{\alpha_S(\mathbf{p}^2)}{2} f(x,\boldsymbol{\Delta})\times|\Psi(z,\mathbf{p}_+)-\Psi(z,\mathbf{p}_+-\boldsymbol{\Delta})|^2\,, \tag{4}$$

where $\Psi(z,\mathbf{p})$ is the $q\bar{q}$ wave function of the photon and $\boldsymbol{\Delta} = \mathbf{p}_+ + \mathbf{p}_-$ is the jet-jet decorrelation momentum.

3 Collective unintegrated nuclear glue

The color dipole-nucleus cross-section [6] $\sigma_A(\mathbf{r}) = 2\int d^2\mathbf{b}[1-\exp(-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b}))]$ where $T(\mathbf{b})$ is the optical the thickness of a nucleus, defines the collective nuclear glue per unit area in the impact parameter space, $\phi(\mathbf{b},x,\boldsymbol{\kappa})$ [10, 1]:

$$\Gamma_A(\mathbf{b},\mathbf{r}) = [1-\exp(-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b}))] = \int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x,\boldsymbol{\kappa})\{1-\exp[i\boldsymbol{\kappa}\mathbf{r}]\}\,. \tag{5}$$

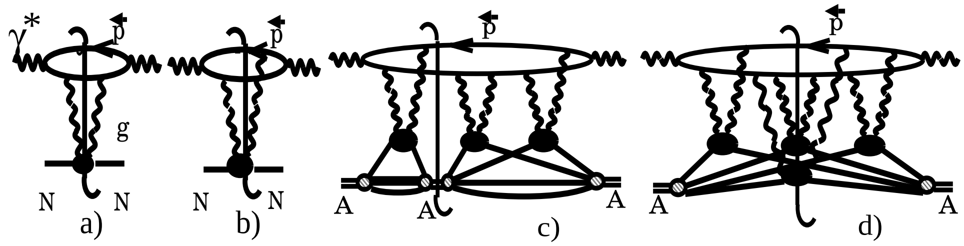

Figure 1: The typical unitarity cuts and dijet final states in DIS : (a),(b) - free-nucleon target, (c) - coherent diffractive DIS off a nucleus, (d) - truly inelastic DIS with multiple color excitation of the nucleus.

A useful expansion is

$$\phi(\mathbf{b},x,\boldsymbol{\kappa}) = \sum_{j=1}^{\infty} w_j(\mathbf{b}) f^{(j)}(x,\boldsymbol{\kappa})\,, \quad w_j(\mathbf{b}) = \frac{1}{j!}\left[\frac{1}{2}T(\mathbf{b})\right]^j \exp\left[-\nu_A(x,\mathbf{b})\right], \tag{6}$$

where $\nu_A(x,\mathbf{b}) = \frac{1}{2}\sigma_0(x)T(\mathbf{b})$, w_j is the probability to find j overlapping nucleons at impact parameter $\mathbf{b}$ in a Lorentz-contracted nucleus and $f^{(j)}(x,\boldsymbol{\kappa})$ is a collective glue of j overlapping nucleons:

$$f^{(j)}(x,\boldsymbol{\kappa}) \;=\; \int \prod_{i=1}^{j} d^2\boldsymbol{\kappa}_i f(x,\boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa} - \sum_{i=1}^{j}\boldsymbol{\kappa}_i) \quad f^{(0)}(x,\boldsymbol{\kappa}) = \delta(\boldsymbol{\kappa}) \tag{7}$$

The plateau at small momenta of gluons,

$$\phi(\mathbf{b},x,\boldsymbol{\kappa}) \approx \frac{1}{\pi}\frac{Q_A^2(\mathbf{b})}{(\boldsymbol{\kappa}^2 + Q_A^2(\mathbf{b}))^2}\,, \quad Q_A^2(\mathbf{b},x) \approx \frac{4\pi^2}{N_c}\alpha_S(Q_A^2)G(x,Q_A^2)T(\mathbf{b})\,, \tag{8}$$

is a signal of the saturation effect. The collective nuclear glue furnishes the linear $k_\perp$-factorization representation for DIS off nuclei (hereafter we focus on $x \lesssim x_A$),

$$\begin{aligned}\sigma_{\gamma^* A} &= \int d^2\mathbf{b}\langle\gamma^*|2\{1-\exp[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})]\}|\gamma^*\rangle \\ &= \int d^2\mathbf{b}\int\frac{d^2\mathbf{p}}{(2\pi)^2}\alpha_S(\mathbf{p}^2)\int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x_A,\boldsymbol{\kappa})(\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa}))^2 \end{aligned} \tag{9}$$

which is the same as for the free-nucleon target, subject to $f(x_A,\boldsymbol{\kappa}) \Longleftrightarrow \phi(\mathbf{b},x_A,\boldsymbol{\kappa})$.

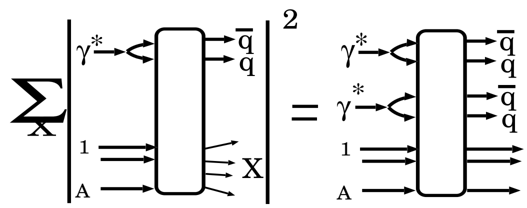

Figure 2: Unitarity diagram for the dijet spectrum in terms of the 4-parton scattering amplitude.

4 The non-abelian intranuclear evolution of color dipoles

The two typical final states in DIS off heavy nucleus are shown in fig. 1. The coherent diffraction with large rapidity gap between the target nucleus in the ground state and diffractive hadronic debris of the photon makes $\approx 50\%$ of the total cross section [11] and gives exactly back-to-back correlated dijets. In the truly inelastic DIS with multiple color excitation of the nucleus one encounters the non-Abelian intranuclear evolution of color dipoles, the consistent description of which based on the ideas from [12, 13] is found in [1] . Specifically, the *ab initio* calculation of the nuclear distortion of the two-parton density matrix the Fourier transform of which gives the spectrum of dijets, can be reduced, upon the closure over nuclear excitations, to the problem of intranuclear propagation of the color-singlet 4-parton states as illustrated in fig. 3:

$$\begin{aligned}
\frac{(2\pi)^4 d\sigma_{in}}{dz d^2\mathbf{p}_+ d^2\mathbf{p}_-} &= \int d^2\mathbf{b}_+{}' d^2\mathbf{b}_-{}' d^2\mathbf{b}_+ d^2\mathbf{b}_- \exp[-i\mathbf{p}_+(\mathbf{b}_+ - \mathbf{b}_+{}') - i\mathbf{p}_-(\mathbf{b}_- - \mathbf{b}_-{}')] \\
&\times \Psi^*(Q^2, z, \mathbf{b}_+{}' - \mathbf{b}_-{}')\Psi(Q^2, z, \mathbf{b}_+ - \mathbf{b}_-) \\
&\times \Big\{S_{4A}(\mathbf{b}_+{}', \mathbf{b}_-{}', \mathbf{b}_+, \mathbf{b}_-) - S_{4A}^{(Diffr)}(\mathbf{b}_+{}', \mathbf{b}_-{}', \mathbf{b}_+, \mathbf{b}_-)\Big\}\,, \qquad (10)
\end{aligned}$$

where we subtracted the diffractive contribution. To the standard dilute-gas nucleus approximation, the Glauber-Gribov theory gives

$$S_{4A}(\mathbf{b}_+{}', \mathbf{b}_-{}', \mathbf{b}_+, \mathbf{b}_-) = \exp\{-\frac{1}{2}\sigma_4(\mathbf{b}_+{}', \mathbf{b}_-{}', \mathbf{b}_+{}', \mathbf{b}_-)T(\mathbf{b})\}\,. \qquad (11)$$

where $\sigma_4(\mathbf{b}_+{}', \mathbf{b}_-{}', \mathbf{b}_+{}', \mathbf{b}_-)$ is the coupled-channel operator in the space of singlet-singlet $|11\rangle$ or octet-octet $|88\rangle$ 4-body dipoles, see ref. [1] for more details.

5 The fate of $k_\perp$-factorization for nuclear targets: the case of DIS

Taken separately, both the truly inelastic,

$$\begin{aligned} \frac{(2\pi)^2 d\sigma_{in}}{d^2\mathbf{b} d^2\mathbf{p} dz} &= \int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, x_A, \boldsymbol{\kappa})\Big|\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa})\Big|^2 \\ &- \Big|\int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, x_A, \boldsymbol{\kappa})(\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa}))\Big|^2 \end{aligned} \tag{12}$$

and coherent diffractive

$$\frac{(2\pi)^2 d\sigma_{D}}{d^2\mathbf{b} d^2\mathbf{p} dz} = \Big|\int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, x_A, \boldsymbol{\kappa})(\Psi(z,\mathbf{p}) - \Psi(z,\mathbf{p}-\boldsymbol{\kappa}))\Big|^2 . \tag{13}$$

spectra are nonlinear functionals of nuclear glue, but they add to precisely the differential form of eq. (9). I.e., the linear $k_\perp$-factorization in terms of the collective nuclear glue holds for total DIS as if there were no Initial and Final State distortions of the spectrum of leading quarks. Such and abelianization is a feature of DIS where the photon is a color singlet projectile, the same is not true for other projectiles, see below Sect. 7.

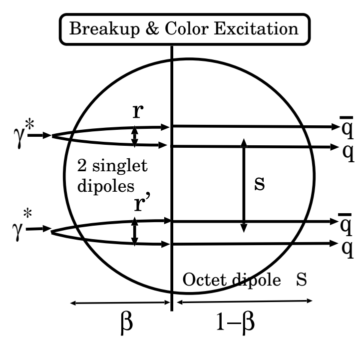

Figure 3: The color excitation of the dipole in the large-N_c approximation.

The nuclear dijet spectrum is a manifestly nonlinear functional of the collective nuclear glue and here emerges a concept of the nonlinear$k_\perp$- factorization(here we cite the

result for the large-N_c approximation):

$$\frac{(2\pi)^2 d\sigma_{in}}{d^2\mathbf{b}dzd\mathbf{p}_-d\mathbf{\Delta}} = \frac{1}{2}T(\mathbf{b})\int_0^1 d\beta \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa} f(x_A,\boldsymbol{\kappa})\Phi((1-\beta)\nu_A(\mathbf{b}),x_A,\mathbf{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})$$
$$\times \quad \Phi((1-\beta)\nu_A(\mathbf{b}),x_A,\boldsymbol{\kappa}_1)\Big|\Psi(\beta;z,\mathbf{p}_-+\boldsymbol{\kappa}_1)-\Psi(\beta;z,\mathbf{p}_-+\boldsymbol{\kappa}_1+\boldsymbol{\kappa})\Big|^2. \qquad (14)$$

where $\Phi(\nu_A(\mathbf{b}),x_A,\boldsymbol{\kappa}) = \delta(\boldsymbol{\kappa})\exp(-\nu_A(\mathbf{b})) + \phi(\mathbf{b},x_A,\boldsymbol{\kappa})$ and

$$\Psi(\beta;z,\mathbf{p}) = \int d^2\boldsymbol{\kappa}\Phi(\beta\nu_A(\mathbf{b}),x_A,\boldsymbol{\kappa})\Psi(\beta;z,\mathbf{p}_-+\boldsymbol{\kappa}) \qquad (15)$$

is the wave function of the incident color-singlet dipole distorted by the Initial State Interaction in the slice β of a nucleus (see fig. 5). The slice $(1-\beta)$ in which the dipole is in the color-octet state gives the Final State Interaction. In DIS it looks as an independent broadening of the quark and antiquark jets. Evidently, eq. (14 entails nuclear enhancement of the decorrelation of jets, the semihard dijets, $|\mathbf{p}_\pm|^2 \lesssim Q_A^2$, are completely decorrelated.

6 The fate of $k_\perp$-factorization for nuclear targets: dijets from pA collisions

Here we comment on the recent generalization of nonlinear $k_\perp$-factorization from DIS to hard interactions of hadrons and nuclei [4]. The pQCD subprocesses relevant to dijet production in pp collisions and the proton hemisphere of pA collisions at RHIC [14, 15] are collisions of a beam quark q^* or a gluon g^* with a gluon from the target, $q^*g \to qg$ or $g^*g \to gg$, respectively. In contrast to the colorless photon in DIS the fragmenting parton is a colored one. We describe the principal changes caused by that on an example of fragmentation $Q^* \to qg$. The total spectrum of dijets including the diffractive component in which the target nucleus remains in the ground state, will be described by eq. (10) in which the last line will be replaced by the combination of multiparton S-matrices shown in fig. 4. The intranuclear propagation of the 2-parton and 3-parton states is a single-channel problem [13, 1], the non-Abelian evolution of the 4-parton state is a three-channel problem, the detailed solution of which will be published elsewhere [4]. Here we

only cite the nuclear dijet spectrum found in the large-N_c approximation:

$$\begin{aligned}
\frac{(2\pi)^2 d\sigma_{in}}{d^2\mathbf{b}dzd\mathbf{p}_g d\boldsymbol{\Delta}} &= \frac{1}{2}T(\mathbf{b})\int_0^1 d\beta \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa} f(x_A,\boldsymbol{\kappa}) \\
&\times \Phi((2-\beta)\nu_A(\mathbf{b}), x_A\boldsymbol{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\Phi((1-\beta)\nu_A(\mathbf{b}), x_A, \boldsymbol{\kappa}_1) \\
&\times \Big|\Psi(\beta; z_g, \mathbf{p}_g+\boldsymbol{\kappa}_1) - -\Psi(\beta; z_g, \mathbf{p}_g+\boldsymbol{\kappa}_1+\boldsymbol{\kappa})\Big|^2 \\
&+ \phi(\nu_A(\mathbf{b}), x_A, \boldsymbol{\Delta})\Big|\Psi(1; z_g, \mathbf{p}_g) - \Psi(z_g, \mathbf{p}_g+z\boldsymbol{\Delta})\Big|^2 \\
&+ \delta(\boldsymbol{\Delta})\Big|\Psi(1; z_g, \mathbf{p}_g) - \Psi(z_g, \mathbf{p}_g)\Big|^2 \exp\left[-\nu_A(\mathbf{b})\right] , \qquad (16)
\end{aligned}$$

where $\Psi(z_g, \mathbf{p}_g)$ is the momentum-space WF of the qg Fock state of the quark Q^*. This

Figure 4: The S-matrix structure of the two-body density matrix for excitation $a \to bc$.

must be compared to the linear $k_\perp$-factorization for the free-nucleon target at large N_c:

$$\begin{aligned}
&\frac{(2\pi)^2 d\sigma_N}{dz d^2\mathbf{p}_+ d^2\boldsymbol{\Delta}} = \frac{\alpha_S(\mathbf{p}^2)}{2} f(x,\boldsymbol{\Delta}) \\
\times \quad &\Big\{\big|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g-\boldsymbol{\Delta})\big|^2 + \big|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+z\boldsymbol{\Delta})\big|^2\Big\}, \qquad (17)
\end{aligned}$$

The third component in (16) is the contribution from the coherent diffractive excitation $q^* \to (qg)A$, which gives the exactly back-to-back dijets. Here the nuclear attenuation factor is a consequence of the initial parton q^* having been colored one. The term in (16) is a counterpart of the second term in the free-nucleon spectrum, it satisfies the linear $k_\perp$-factorization in terms of $\phi(\boldsymbol{\Delta})$ Finally, the first component of the free nucleon spectrum (17) gives rise to the nonlinear $k_\perp$-factorization component in the nuclear spectrum (16), which resembles strongly the truly inelastic dijet spectrum (14) for DIS. The principal difference is in the nuclear thickness dependence of the distortion factor: the

asymmetric one, $\Phi((2-\beta)\nu_A(\mathbf{b}), x_A, \mathbf{\Delta} - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa})\Phi((1-\beta)\nu_A(\mathbf{b}), x_A, \boldsymbol{\kappa}_1)$ for the fragmentation of colored quark q^* vs. the symmetric one, $\Phi((1-\beta)\nu_A(\mathbf{b}), x_A, \mathbf{\Delta} - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa})\Phi((1-\beta)\nu_A(\mathbf{b}), x_A, \boldsymbol{\kappa}_1)$ in DIS. In DIS it describes equal distortion of the both outgoing parton waves by pure FSI, for the incident quarks q^* in pA collisions it includes the ISI distortion of the incoming wave of the colored quark q^*. Subject to slight modifications for the color-representation dependence of the collective nuclear glue, the decomposition (16) of the nuclear dijet spectrum will hold for dijet production via other pQCD subprocesses like heavy flavour excitation $g^* \to Q\bar{Q}$ or gluon splitting $g^* \to gg$.

7 The fate of $k_\perp$-factorization for nuclear targets: single-jet spectra in pA collisions

The recovery of linear $k_\perp$-factorization (9) for the single-jet spectrum in DIS is rather an exception due to the abelianization in the case of a colorless projectile - the photon. The radiation of gluons from quarks, $q^* \to qg$, illustrates nicely the salient features of breaking of linear $k_\perp$-factorization for the single-jet spectrum [3]. It is directly relevant to jet production in the proton hemisphere of pA collisions at RHIC [14, 15].

Here we again show the large-N_c results. The spectrum of gluons for the free-nucleon target reads

$$\begin{aligned}
&\frac{(2\pi)^2 d\sigma_A(q^* \to g(\mathbf{p}_g)q)}{dz_g d^2\mathbf{p}_g} = \frac{1}{2}\int d^2\boldsymbol{\kappa} f(x,\boldsymbol{\kappa}) \\
&\left\{\left|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa})\right|^2 + \left|\Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}) - \Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa})\right|^2\right\}. \qquad (18)
\end{aligned}$$

The same spectrum for the nuclear target is of the two-component form

$$\begin{aligned}
&\frac{(2\pi)^2 d\sigma_A(q^* \to g(\mathbf{p}_g)q)}{dz_g d^2\mathbf{p}_g d^2\mathbf{b}} = \exp\left[-\nu_A(\mathbf{b})\right]\int d^2\boldsymbol{\kappa}\phi(\mathbf{b},x_A,\boldsymbol{\kappa}) \\
&\left\{|\Psi(z_g,\mathbf{p}_g) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa})|^2 + |\Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}) - \Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa})|^2\right\} \qquad (19) \\
&+ \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \phi(\mathbf{b},x_A,\boldsymbol{\kappa}_1)\phi(\mathbf{b},x_A,\boldsymbol{\kappa}_2)|\Psi(z_g,\mathbf{p}_g+z_g\boldsymbol{\kappa}_1) - \Psi(z_g,\mathbf{p}_g+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)|^2 .
\end{aligned}$$

The first component is an exact counterpart of the free-nucleon spectrum subject to the familiar substitution $f(x_A,\boldsymbol{\kappa}) \to \phi(\mathbf{b},x_A,\boldsymbol{\kappa})$. It is suppressed by the nuclear absorption factor and for central interactions the spectrum is entirely dominated by the second component which is a non-linear functional of the collective nuclear glue.

For soft gluons , $z_g \ll 1$, the result (20) simplifies to

$$\frac{(2\pi)^2 d\sigma_A(q^* \to g(\mathbf{p}_g)q)}{dz_g d^2\mathbf{p}_g d^2\mathbf{b}} = \int d^2\boldsymbol{\kappa}\, \phi_{gg}(\mathbf{b}, x_A, \boldsymbol{\kappa})|\Psi(z_g, \mathbf{p}_g) - \Psi(z_g, \mathbf{p}_g + \boldsymbol{\kappa})|^2 . \tag{20}$$

In takes the linear $k_\perp$-factorization form in terms of $\phi_{gg}(\mathbf{b}, x_A, \boldsymbol{\kappa}) = \Big(\phi \otimes \phi\Big)(\mathbf{b}, x_A, \boldsymbol{\kappa})$ which has a meaning of the collective nuclear glue defined in terms of the intranuclear propagation of the gluon-gluon color dipole. This illustrates nicely the important point that the collective nuclear glue is a density matrix in the color space rather than a single scalar function [1].

One more point is noteworthy: there is a conspicuous difference between the z_g-dependence of the free-nucleon and nuclear spectra. This amounts to the $\mathbf{p}_g$-dependence of the Landau-Pomeranchuk-Migdal effect; the same applies to the spectrum of leading quarks and nuclear quenching of forward jets in pA collisions [3].

8 Small-x evolution of collective nuclear glue.

Despite the manifest breaking of the linear $k_\perp$-factorization, the collective nuclear glue remains a useful concept. For a free nucleon target the effect of the $q\bar{q}g$ and higher Fock states in the photon is reabsorbed in the linear BFKL evolution for the dipole cross section with the photon treated as the $q\bar{q}$ state. One possible definition of the nonlinear BFKL evolution for the nuclear unintegrated glue is to insist on the same representation for the nuclear profile function $\Gamma_A(\mathbf{b}, x, \mathbf{r})$. It is indeed possible although without a closed-form evolution equation.

We comment here on the first iteration of the $\log\frac{1}{x}$ evolution. The modification of the color dipole-nucleus profile function for the $q\bar{q}g$ Fock state in the photon equals

$$\int d^2\mathbf{b} \frac{\partial \delta\Gamma_A(\mathbf{b}, x, \mathbf{r})}{\partial \log\frac{1}{x}} = K_0 \int d^2\boldsymbol{\rho} \frac{\mathbf{r}^2}{\boldsymbol{\rho}^2(\boldsymbol{\rho} - \mathbf{r})^2} 2 \int d^2\mathbf{b} [\Gamma_{3A}(\mathbf{b}, \boldsymbol{\rho}, \mathbf{r}) - \Gamma_{2A}(\mathbf{b}, \mathbf{r})] \tag{21}$$

$$\Gamma_{3A}(\mathbf{b}, \boldsymbol{\rho}, \mathbf{r}) \quad = \quad 1 - S_{3A}(\mathbf{b}, \boldsymbol{\rho}, \mathbf{r}) = 1 - \exp[-\frac{1}{2}\sigma_1(\boldsymbol{\rho}, \mathbf{r})T(\mathbf{b})] \tag{22}$$

where $\sigma_3(\boldsymbol{\rho}, \mathbf{r})$ is the 3-parton cross section [8].

A simplified Glauber-Gribov formula holds at large-N_c, $S_{3A}(\mathbf{b}, \boldsymbol{\rho}, \mathbf{r}) = S_{2A}(\mathbf{b}, \boldsymbol{\rho} - \mathbf{r})S_{2A}(\mathbf{b}, \boldsymbol{\rho})$. Here $\partial\Gamma_A(\mathbf{b}, x, \mathbf{r})/\partial \log\frac{1}{x}$ is a nonlinear functional of Γ_{2A}, the identification of $\Gamma_A(x, \mathbf{b}, \mathbf{r})$ with $\Gamma_{2A}(x, \mathbf{b}, \mathbf{r})$, and the extension of the first iteration to the closed-form

nonlinear equation as claimed in ref. [16] is unwarranted. In terms of the nuclear transparency for large dipoles, $S_A(\mathbf{b},\sigma_0) = \exp[-\nu_A(\mathbf{b})]$, the first iteration for unintegrated nuclear glue takes the form

$$\begin{aligned} \frac{\partial \delta\phi_A(\mathbf{b},x,\mathbf{\Delta})}{\partial \log\frac{1}{x}} &= S_A(\mathbf{b},\sigma_0)\mathcal{K}_{BFKL}\otimes\phi(\mathbf{b},x_A,\mathbf{\Delta}) \\ &+ K_0\int d^2\mathbf{p}d^2\mathbf{k}\phi(\mathbf{b},x_A,\mathbf{k}) \\ &\times \Big\{K(\mathbf{\Delta}+\mathbf{p},\mathbf{\Delta}+\mathbf{k})\phi(\mathbf{b},x_A,\mathbf{p}) - K(\mathbf{p},\mathbf{p}+\mathbf{\Delta}+\mathbf{k})\phi(\mathbf{b},x_A,\mathbf{\Delta})\Big\} \\ &= S_A(\mathbf{b},\sigma_0)\mathcal{K}_{BFKL}\otimes\phi(\mathbf{b},x_A,\mathbf{\Delta}) + \mathcal{K}_{NonLin}\big[\phi(\mathbf{b},x_A,\mathbf{\Delta})\big] \end{aligned} \quad (23)$$

where $K(\mathbf{p},\mathbf{k}) = (\mathbf{p}-\mathbf{k})^2/\mathbf{p}^2\mathbf{k}^2$. It contains an absorption suppressed linear BFKL term with the familiar kernel $\mathcal{K}_{BFKL}$ [17]. For central DIS off heavy nuclei $S_A \to 0$ and evolution is entirely driven by the nonlinear term quadratic in $\phi(\mathbf{b},x_A,\mathbf{k})$.

Making use of an explicit form of $K(\mathbf{p},\mathbf{k})$, one can recast (23) for the leading conformal twist nuclear glue in an alternative form

$$\frac{\partial\phi(\mathbf{\Delta},x\mathbf{b})}{\partial\log(1/x)} = \mathcal{K}_{BFKL}\otimes\phi(\mathbf{b},x_A,\mathbf{\Delta}) + \mathcal{Q}[\phi](\mathbf{b},x_A,\mathbf{\Delta})\,. \quad (24)$$

Here the linear term evolves with the conventional BFKL kernel, whereas the nonlinear term takes a particularly simple form

$$\begin{aligned} \mathcal{Q}[\phi](\mathbf{b},x,\mathbf{\Delta}) = &- \frac{2K_0}{\mathbf{\Delta}^2}\Big[\int_{\mathbf{\Delta}^2} d^2\mathbf{q}\phi(\mathbf{b},x_A,\mathbf{q})\Big]^2 \\ &- 2K_0\phi(\mathbf{b},x_A,\mathbf{\Delta})\int_{\mathbf{\Delta}^2}\frac{d^2\mathbf{p}}{\mathbf{p}^2}\int_{\mathbf{p}^2} d^2\mathbf{q}\phi(\mathbf{b},x_A,\mathbf{q}). \end{aligned} \quad (25)$$

For hard gluons, $\mathbf{\Delta}^2 > Q_A^2$, one can use an approximation $\phi(\mathbf{q}) \sim \phi(\mathbf{\Delta})\left(\mathbf{\Delta}^2/q^2\right)^2$ with the result

$$\mathcal{Q}[\phi](\mathbf{\Delta};\mathbf{b}) \approx -4K_0\cdot\mathbf{\Delta}^2\phi^2(\mathbf{\Delta}) \propto \frac{\phi(\mathbf{\Delta})}{\mathbf{\Delta}^2}\,. \quad (26)$$

The nonlinear component in (24) gives a pure higher twist contribution. It doesn't exhaust the nuclear higher twist terms, though, because the one is contained also in $\phi(x,\mathbf{\Delta};\mathbf{b})$, see the discussion in [1, 10]. The character of nonlinearity in terms of $G_A(\mathbf{b},x_A,Q)$ is instructive:

$$\frac{\partial^2\delta G_A(\mathbf{b},x,Q)}{\partial\log(1/x)\partial\log Q^2} = \mathcal{K}_{BFKL}\otimes\frac{\partial G_A(\mathbf{b},x_A,Q)}{\partial\log Q^2} - \frac{4\alpha_S(Q^2)T(\mathbf{b})}{Q^2}\cdot\left(\frac{\partial G_A(\mathbf{b},x_A,Q)}{\partial\log Q^2}\right)^2 \quad (27)$$

Now have a look at the plateau region of soft gluons , $\mathbf{\Delta}^2 \ll Q_A^2$. Here eq. (23) takes the form

$$\frac{\partial \phi_A(\mathbf{b}, x, \mathbf{\Delta})}{\partial \log \frac{1}{x}} = -2C\pi K_0 \phi(\mathbf{b}, x_A, 0) \tag{28}$$

where the constant factor, $C \sim 1$, depends on the form of the collective nuclear glue. If we recall that

$$\phi(x, \mathbf{b}, 0) \sim \frac{1}{\pi Q_A^2(\mathbf{b}, x)} \tag{29}$$

then (28) entails an expansion of the plateau width with the decrease of x:

$$Q_A^2(\mathbf{b}) \Longrightarrow Q_A^2(\mathbf{b}) \left[1 + 2C\pi K_0 \log \frac{1}{x}\right] \tag{30}$$

The full-fledged nonlinear evolution will be in effect for soft-to-hard intermediate gluon momenta $\mathbf{\Delta}^2 \lesssim Q_A^2$.

9 CONCLUSIONS

Nuclear saturation is a straightforward consequence of opacity of heavy nuclei to large color dipoles. The imposition of unitarity constraints within the color-dipole approach leads to a unique definition and expansion of nuclear unintegrated glue in terms of *the collective glue of overlapping nucleons*. The problem of *a non-abelian* intranuclear evolution of color dipoles has been solved and a consistent description of single-jet and dijet production in DIS off nuclei and hadron-nucleus collisions has been developed. We have proven the *breaking* $k_\perp$ *factorization* and instead formulated the *nonlinear* $k_\perp$*-factorization* for forward single-jet and dijet production. The *nonlinear* $k_\perp$*-factorization* emerges as a universal feature of the pQCD description of hard scattering in nuclear environment, still its mathematical formulation depends on the relevant pQCD subprocess. We applied our technique to the nonlinear BFKL evolution of collective nuclear glue and explored the twist properties of the nonlinear component of this equation.

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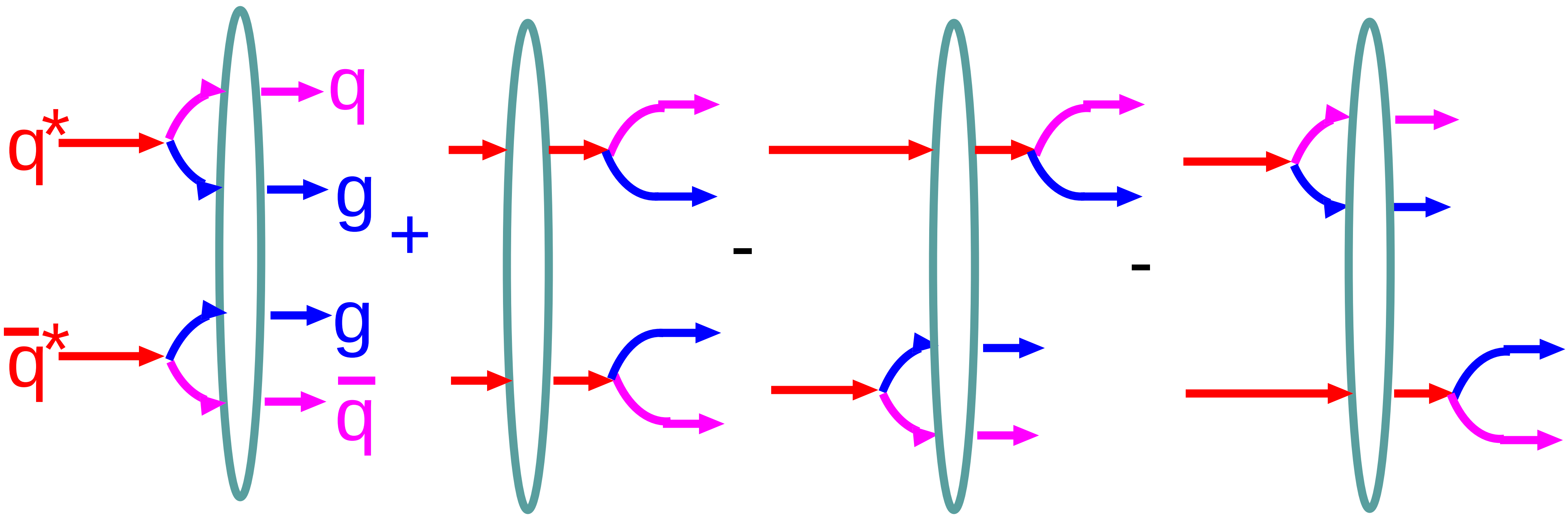

$$S^{(4)}_{\bar{q}gqg} + S^{(2)}_{\bar{q}^*q^*} - S^{(3)}_{q^*g\bar{q}} - S^{(3)}_{\bar{q}^*gq}$$